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Title: House prices in Denmark: are they far from equilibrium?[1]

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Abstract:

House prices and investments increased at a fast pace in Denmark until the mid 2000s. Following interest rate hikes through end-2008 and the escalation of the financial and economic crisis, housing market developments went into a sharp reverse, but now appears to have stabilised. To explain these developments we derive and estimate a traditional demand-supply housing model. Our multivariate econometric analyses and simulations indicates that house price developments during the upturn were in line with fundamentals and the unusually strong price appreciation between 2004 and 2007 was propped up by mortgage rates falling well below historical levels. Moreover, the price peak in the recent upturn appears not to be overly different from previous upturns when measured relative to an estimated equilibrium value. We find that both house prices and investments mid 2010 were close to their estimated equilibrium value. Given plausible scenarios for mortgage rates and economic activity our model predicts that real house prices continues to stabilize during 2010 for then to start appreciating, while housing investments may soon start to bounce back at a non trivial pace.

[1] The views expressed in the working paper are those of the authors, not necessarily the Ministry of Finance.

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1. Introduction

After a more than decade-long period of brisk growth house prices declined rapidly between early 2007 to mid 2009 but have since then stabilized. Until recently, declining house prices were particularly evident for apartments and single-family homes in the greater Copenhagen area where past excesses were being reversed at high speed. This followed real house price increases until 2007 that were large (and lasted for a long time) both in an international and a historic perspective. However, the sizeable increases came on the back of a highly depressed housing market in the early to mid 1990s.

While there is little doubt that the house price increases at the end of the upturn were unsustainable, there is uncertainty as to whether the subsequent price declines has returned house price to a natural level. To examine this, we set-up and estimate a traditional demand-supply housing model using multivariate co-integration techniques so to capture feed-back effects between long-run relationships. We find two long run co-integrating relationships: a demand equation with house prices determined by disposable income, financial wealth, housing stock and user costs; and a traditional Tobin's q supply equation for housing investments.

Simulations of our estimated model suggests that even though the recent house price appreciation was extraordinary rapid it was in general supported by fundamentals such as falling real interest rates, a nominal tax freeze on property value taxes, sharply falling unemployment rates, and strong growth in disposable income and financial wealth. The introduction of new loan forms, such as adjustable-rate and interest-only mortgages added to these trends.

Our model also suggests house prices were close to (and housing investments below) an estimated equilibrium value mid 2010, indicating that they may now have found a more natural level. To examine housing market trends going forward we establish two scenarios differing mainly in their future path for mortgage rates. Both scenarios imply real house prices continuing to stabilize through 2010 and then rebounding during 2011-12 despite an assumed gradual monetary policy tightening beginning mid 2011. Meanwhile, real housing investments may start to increase already in the second half of 2010.

The paper begins by reviewing recent house price trends. It then sets up a theoretical framework for house prices. The following section uses multivariate econometric techniques to establish long run relationships for house prices and investments. The subsequent section looks at housing market perspectives going forward using our estimated model to simulate possible paths for house prices and investments. A few observations summarises the paper.

2. The Housing Market: Some Empirical Trends

House price trends

Strong economic fundamentals over the 10 to 15 years leading up to the financial crisis sparked an unprecedented real house price appreciation from the mid 1990s through mid 2000s. The combination of falling real interest rates, a nominal freeze of property value taxes, sharply falling unemployment rates, robust disposable income growth, sound fiscal and monetary policies, and stock market gains provided a powerful boost to the financial situation and sense of security of households. The introduction of new loan forms such as adjustable-rate and interest-only mortgages have further supported these strong fundamentals. However, triggered by mortgage rate hikes through end 2008 and subsequently the financial and economic crisis, real house prices declined rapidly until mid 2009 and may now have stabilized at a more sustainable level.

The housing market has gone through a number of up and down turns over the past 40 years, but the recent upturn appears unprecedented in length and real price increase (see Table 2.1 and Figure 2.1). While previous up- and downturns have lasted between 13 and 18 quarters (3 to 5 years), the latest upturn lasted 55 quarters (14 years) and exhibited a real price increase of 177 percent (or 146 percent relative to income), although real price increases between 1998 and 2003 were fairly mild. In the previous two upturns real price increases were more modest. The real price decline witnessed so far in the current price downturn amounts to more than 20 percent, corresponding to two-thirds of previous real price declines, but on the back of a steeper increase. However, relative to households' disposable income the price increases in the recent upturn look less dramatic and the recent correction is of a similar magnitude as previous downward corrections. Furthermore, the strong upturn came on the back of a protracted housing slump from mid 1980s to early 1990s.

Table 2.1. House price changes

Peak	Trough	Peak	No of quarters	Real price change	Price to income change
Country-wide					
	1974:Q3	1979:Q2	18	25,6	5,4
1979:Q2	1982:Q4		14	-34,7	-41,9
	1982:Q4	1986:Q1	13	60,0	94,1
1986:Q1	1993:Q2		29	-32,2	-49,0
	1993:Q2	2007:Q1	55	176,6	145,5
2007:Q1	2009:Q3		9	-22,0	-44,3
	2009:Q3	2010:Q1	2	1,2	0,7
Copenhagen (apartments)					
1986:Q1	1993:Q2		29	-43,9	-58,7
	1993:Q2	2006:Q2	52	411,1	376,6
2006:Q2	2009:Q2		12	-39,8	-58,9
	2009:Q2	2010:Q2	4	9,9	8,6

Source: Statistics Denmark, Nationalbanken, and Realkreditrådet

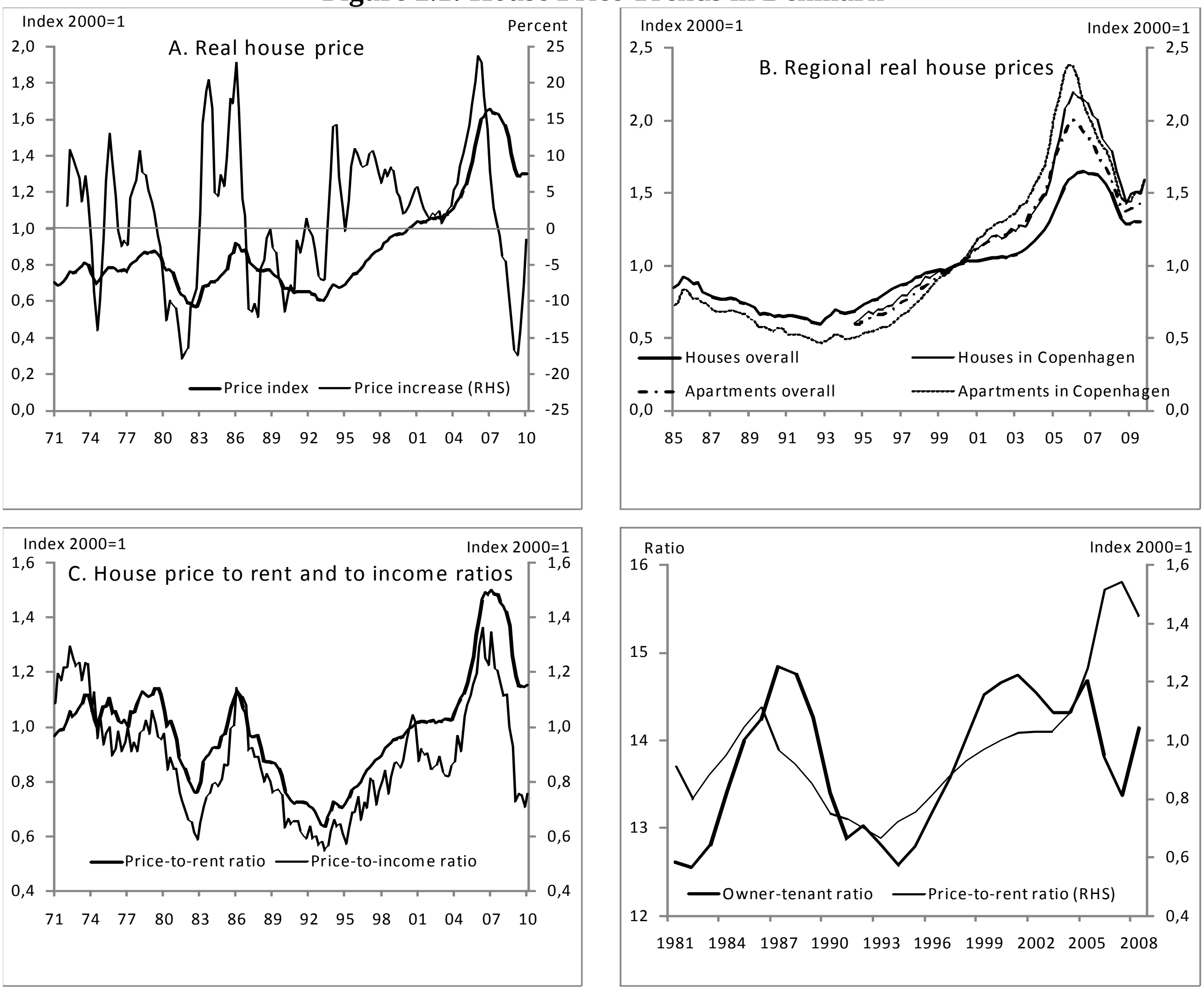
Note: Troughs and peaks are based on the real price developments.

Income is household disposable income.

House prices in metropolitan Copenhagen have exhibited stronger volatility than at national level (see Figure 2.1). This is even more so for apartments, which is a relatively large segment in Copenhagen. Real price increases for apartments expanded by more than 400 percent over a 13 year period until mid 2006. Since then, they have fallen by 40 percent (reaching their early 2004 level), but are now bouncing back. The more pronounced boom and bust phase in the Copenhagen housing market may indicate less elastic supply and new innovations such as parents buying apartments for their children etc. However, it may also partly be a result of more than average marketed cycle in the real economy. For instance, between 1994 and 2008 unemployment rates declined 14 percentage points in Copenhagen compared to 10 percentage points on average.

Empirical findings show that deviations of the house price-to-rent ratio from its equilibrium are usually not sustained for extended periods (Leamer, 2002). A caveat, though, is that the Danish rental market is highly regulated and therefore rents may not fully reflect market prices. As a result of the recent declines, house prices relative to rents and also disposable incomes are now at significantly lower levels. The price-to-rent ratio was still above the historical average in early 2010, while importantly the price-to-disposable income ratio was more than 10 percent below. Thus, both indicators points to a more normalised housing market.

Figure 2.1: House Price Trends in Denmark



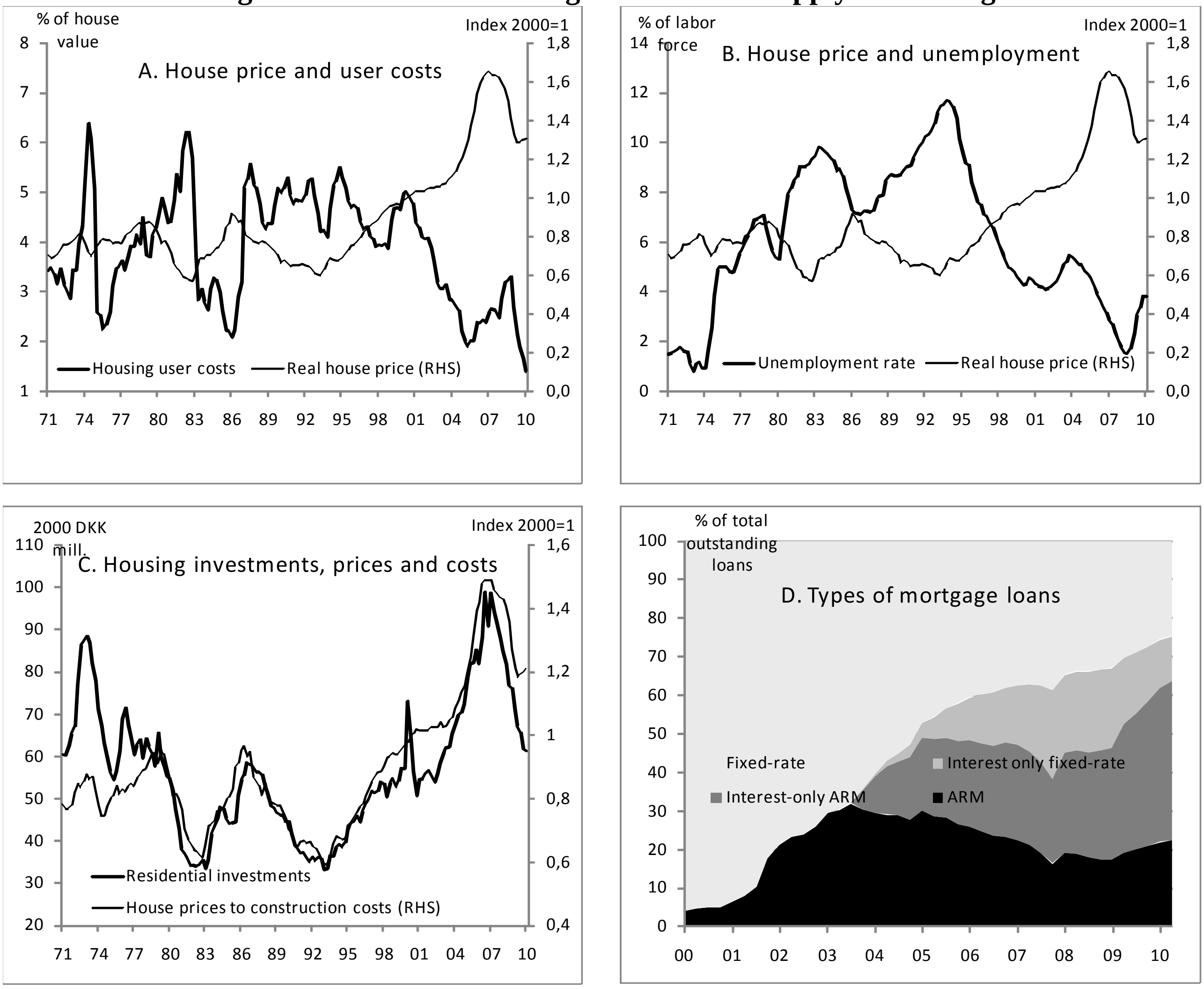
Source: The Nationalbank, Statistics Denmark and Danish tax authorities (SKAT).

Factors affecting the demand and supply of housing

The average user costs of holding a house declined steadily between the mid 1990s and mid 2000s on the back of declining mortgage rates, increased use of adjustable-rate mortgages, and falling property-related tax rates (Figure 2.2). A phased reduction in the mortgage rate income tax deductibility (from 46 percent in 1998 to 33 percent in 2001) offset some of the forces lowering housing user costs. Nevertheless, lower after tax real mortgage rates improved household's debt service capacity and user costs are now only a third of what they used to be during the 1990s.

The introduction of adjustable-rate and interest-only mortgages has underpinned the strong real price growth. Before 1997/98 no homeowners had adjustable rate mortgages, whereas today they constitute almost two-thirds of all mortgages. Similarly, since the introduction of interest-only mortgages at end 2003 the share of those loans has expanded to more than half of all mortgage loans and the share of homeowners with an adjustable rate interest-only mortgage was more than 40 percent at mid 2010. The latter figure may in itself pose a risk if the interest rate suddenly increases fast and if those mortgage holders face a tight budget situation.

Figure 2.2: Factors Affecting Demand and Supply of Housing



Source: The Nationalbank, Statistics Denmark, and Realkreditrådet.

The sustained price hikes have also been associated with major reductions in the unemployment rate that topped at 12 percent in the mid 1990s and went below 2 percent in 2008. This sharp decline may have instilled a greater sense of job-security and thereby led to less risk-aversion among households and financial institutions. As such, beyond the normal effect of rising incomes, lower unemployment levels may itself have sparked larger house price appreciation.

On the supply side, housing investments seem highly responsive to changes in the price of a house relative to the costs of construction a new one (see Figure 2.3C). During the upturn they increase by roughly 200 percent in real terms, but have since then declined by almost 40 percent and as a share of GDP is now 1 percentage point below the historical average (or at the historical average if the investment binge in the 1970s is excluded).

Demographic trends and individual behaviour may have reinforced housing demand. Homeownership has increased by more than 250,000 over the past 30 years as a result of increasing population size and higher ownership rates among the +45 year olds.[3] However, homeownership rates have declined for each 5-year cohort among the 20-45 year olds. Declining household size has also bolstered demand as well as a tendency to own a larger home (size in square meters). What also matters is an unambiguous trend increase in demand for higher quality housing as seen by the increase in houses with central heating, bath and toilet (see Table 2.2).

Table 2.3: Demand for Housing

	1981	1990	2000	2008
Household size (persons)				
Houses, all country	2.9	2.6	2.5	2.4
Apartments, all country	1.9	1.7	1.7	1.7
Apartments, Copenhage	1.7	1.7	1.7	1.8
Home size (square meters)				
Houses, all country	132	135	138	142
Apartments, all country	75	76	76	77
Apartments, Copenhage	69	72	74	76
Homes with central heating, bath and toilet (percentage)				
Houses, all country	91	94	96	97
Apartments, all country	71	79	89	93
Apartments, Copenhage	55	65	79	88

Source: Statistics Denmark

3. Modelling Framework

In this paper we model house prices using a standard housing demand-supply model based on households maximising an inter-temporal utility function with non-separability between housing and non housing consumption. As a supply response firms maximise profits by investing and building housing units. The model takes into account inter and intra temporal substitution between housing and non-housing consumption.

Demand side

Following work by Aoki et al. (2002), Ayuso and Restoy (2006), Miles (1994) and Poterba (1984) the representative household in our model derives utility from consumption of a composite good, c_t, and housing services, h_t. The representative household maximises its utility function in all future periods discounted by the rate of time preference, β:[4]

$$\max_{c,h} E_0 \sum_{t=0}^{\infty} (1+\beta)^{-t} u(c_t, h_t) \qquad (3.1)$$

The representative household is subject to a budget constraint that holds each period:

$$W_t = (1+r)W_{t-1} + y_t - c_t - p_t h_t$$

[3] Mankiw and Weil (1989) show for the United States that there exist a significantly positive relationship between demographic induced demand and house prices.
[4] See appendix A.1 for thorough calculations.

Where W_t is real financial wealth (including housing equity), r is the constant real riskless return on the equilibrium asset portfolio, y_t is real endowment income, and p_t is the real house price (the price of c_t is numeriare). Real housing consumption, h_t, equals the user cost of housing, uc_t, times the housing stock, *H*; $h_t = uc_t \cdot H_t$. Given the budget constraint we can solve the representative household's maximisation problem via the usual Lagrange function. The first-order conditions to the maximisation problem imply the following relationships between marginal utilities u_c' and u_H':

$$L'_c = (1+\beta)^{-t} u_c'(c_t, h_t) - \lambda(1+r)^{-t} = 0$$

$$L'_h = (1+\beta)^{-t} u_h'(c_t, h_t) - \lambda(1+r)^{-t} \cdot p_t = 0$$

Dividing the two first order conditions with each other gives the usual intra-temporal relationship between marginal utilities showing that the marginal rate of substitution (MRS) between non-housing and housing consumption is equal to the relative prices (since the price of c_t is numeraire):

$$\frac{u_h'(c_t, h_t)}{u_c'(c_t, h_t)} = p_t \qquad (3.2)$$

As the representative consumer also maximizes utility across time we calculate the development of household consumption over time. The standard Euler-equations can be derived from the first order conditions by dividing the conditions in two periods:

$$\frac{u_c'(c_t, h_t)}{u_c'(c_{t+1}, h_{t+1})} = \frac{1+r}{1+\beta} \; and \; \frac{u_h'(c_t, h_t)}{u_h'(c_{t+1}, h_{t+1})} = \frac{1+r}{1+\beta}(1+\pi_t) \qquad (3.3)$$

Where π_t is the real house price growth rate. We assume $u(c_t, h_t) = \frac{\gamma \cdot c_t^{1-\theta} + (1-\gamma) \cdot h_t^{1-\theta}}{1-\theta}$ as the functional form for the utility function and by inserting in the MRS equation (3.2) we get the following expression for house prices:

$$p_t = \frac{1-\gamma}{\gamma} \cdot \left(\frac{c_t}{h_t}\right)^{\theta} \qquad (3.4)$$

Recalling that $h_t = uc_t \cdot H_t$ and taking the logarithm of (3.4) yields:

$$\log p_t = \theta \cdot (\log c_t - \log uc_t - \log H_t) + \log\left(\frac{1-\gamma}{\gamma}\right) \qquad (3.5)$$

By using the functional form for the utility function we can rewrite the Euler-equations (3.3) and by incorporating into the budget constraint we arrive at:

$$c_0 \sum_{t=0}^{\infty} \left(\frac{1+r}{1+\beta}\right)^{\frac{1}{\theta}t} (1+r)^{-t} = W_0 + \sum_{t=0}^{\infty} (1+r)^{-t} y_t - h_0 \sum_{t=0}^{\infty} \left(\frac{(1+r)(1+\pi_t)}{1+\beta}\right)^{\frac{1}{\theta}t} (1+r)^{-t} p_t$$

Rearranging into a simpler expression and inserting into equation (3.5) gives us the demand side house price equation:

$$\log p_0 = \sigma \log W_0 + \omega \log \hat{y}_0 - \rho(\log H_0 + \log uc_0) + \vartheta_0 \qquad (3.6)$$

The equation shows the standard result that house prices depend negatively on user costs and housing stock, while positively on wealth and disposable income, see Barot and Yang (2002).

Supply side
To model the supply side we follow Brøchner (1992) and Poterba (1984) who use an intertemporal capital stock approach where the representative firm chooses the optimal amount of investment by maximising the net present value of profits:

$$\max_{H,I} \Pi = \sum_{t=0}^{\infty}(1+r)^{-t}[p_t H_t - pi_t I_t - C(I,H)] \qquad (3.7)$$

The representative firm maximises subject to the evolution of housing stock:

$$H_{t+1} = (1-\delta)H_t + I_t$$

Profits are given by the value of housing, $p_t^H H_t$, minus the costs of investment which is divided into direct investment costs, $p_t^I I_t$, and some convex adjustment costs, $C(I,H) = ½\beta\frac{I^2}{H}$. The latter functional form is homogenous of degree one. The investment costs are strictly convex in I: $C_I{}'(I,H) > 0$, $C_I{}''(I,H) > 0$ and $C_H{}'(I,H) < 0$. Given the budget constraint we can solve the representative firm's maximisation problem via the usual Lagrange function. Solving the problem yields the first order conditions:

$$L_I = (1+r)^{-t}(-pi_t - C_I'(I,H) + q_t) = 0 \qquad (3.8)$$

$$L_H = (1+r)^{-t}\left(p_t - C_H'(I,H) + q_t(1-\delta)\right) - (1-r)^{-(t-1)}q_{t-1} = 0 \qquad (3.9)$$

$$lim(1+r)^{-t}q^t H^t = 0 \qquad (3.10)$$

Equation (3.8) states that the representative firm invests to the point where the price of investment (direct and adjustment costs) equals the value of housing (i.e. the shadow price q_t is equal to the market value of houses relative to the replacement value). Equation (3.10) is the standard transversality condition stating that the present value of the housing stock at infinity is zero.

From the first order conditions it is possible to derive the firms' optimal amount of investment. By solving equation (3.9) forward and solving for q_0 we get the following expression.

$$q_0 = \frac{1}{1-\delta}\sum_{t=0}^{\infty}\left(\frac{1-\delta}{1+r}\right)^t\left(p_t^H - C_H'(I,H)\right) \qquad (3.11)$$

Inserting (3.11) into (3.8), solving for the investment intensity, I/H, taking the logarithm and log-linearize around 1 we get an expression for the firm's optimal investment intensity:

$$log\left(\frac{I_0}{H_0}\right) = \frac{1}{\varphi}\left(\rho + \frac{\omega}{\omega\ \ 1}log(p_0^H) - \frac{1}{\omega\ \ 1}log(p_0^I)\right) \qquad (3.12)$$

Where $\rho = \log\left(\frac{\mu}{1-\delta} - 1\right) - log(\beta) - log\left(1 - \frac{\sigma}{2(1-\delta)}\right)$, $\omega = \frac{\mu}{1-\delta} > 1$and $\varphi = 1 - \frac{1}{\frac{2(1-\delta)}{\sigma} - 1} > 0$.

Equation (3.12) is the supply function (from Tobin's q) in our modelling framework and it states that housing investment relative to the housing stock depends positively on house prices and negatively on investment costs.

4. Statistical Analysis and Results

In this section we estimate the model outlined in the previous section. As the model contains simultaneously determined equations we estimate it using the standard VAR-procedure following Johansen (1990). The *first* step in the modelling approach examines the time series properties of the univariate series. We look at patterns and trends in the data and test for stationarity and the order of integration. *Second*, we form a Vector Autoregressive Regression (VAR) system. This step involves testing for the appropriate lag length, including residual diagnostic tests and tests for model stability. *Third*, we test the system for potential cointegration relationship(s). Data series integrated of the same order may be combined to form economically meaningful series that are integrated of lower order. *Fourth*, we interpret the cointegrating relations and test for weak exogeneity. Based on these results a conditional error correction model of the endogenous variables may be specified, further reduction tests are performed and economic hypotheses tested.

Data and Univariate Analysis

We mainly use data from the Danish Central Banks database, MONA, to estimate the two models. The data set spans 1971-2009 on a quarterly basis and contains information of house prices, housing investments, income, interest rates etc. (for a complete list of variables see appendix A.2).

To examine the properties of the univariate time series we have performed the standard augmented Dickey-Fuller (ADF) test in both levels and differences (see Table 4.1). For almost all variables in levels we cannot reject the null hypothesis of a unit root (except for user costs) which indicate that these follow an I(1) process. Looking at the results for the variables in their differences we see that we can reject the null hypothesis of unit root for all variables in differences. Based on these results we conclude that the variables seem to follow an I(1) process and hence are stationary in their first differences. This is also the conclusion from ADF-tests including a trend.

Table 4.1. Augmented Dickey-Fuller test for model with constant, 1973:1 to 2009:4

	Level		First differences	
	Lag-length	t-ADF	Lag-length	t-ADF
Real house prices (P)	1	-1.064	0	-5,777**
Real housing investments (I)	1	-1.548	0	-9,745**
Real disposable income (Y)	3	0.459	2	-9,015**
Real housing stock (H)	2	-1.394	1	-12,57**
Real financial wealth (W)	6	-0.755	4	-6,012**
User costs (UC)	3	-2,816	5	-6,876**
Real building costs (PI)	1	-0.772	0	-14,02**

Note: All variables are expressed in natural logarithms except for user costs. We use the Akaike information criterion to determine the appropriate lag length.
Source: The Nationalbank and Statistics Denmark

These results do seem to be in line with the fact that many macroeconomic- and aggregate level series are shown to be integrated of order one, or I(1), see Nelson and Plosser (1982). Simple first differencing of the data will remove the non-stationarity problem, but will also cause a loss of information regarding the long run "equilibrium" relationships among the variables, see Davidson *et. al.* (1978). In order to preserve the information about potential long run relations between the variables we estimate the model using the Johansen cointegration procedure, see Johansen (1990).

Empirical methodology
The Johansen procedure examines whether there exists economically meaningful linear combinations of the I(1) series that are stationary. The procedure starts out by specifying an unrestricted Vector Auto Regressive (VAR) system:

$$Y_t = \pi_0 + \sum_{i=1}^{p} \theta_i Y_{t-i} + \psi D_t + e_t$$

Y_t is $(n \; x \; 1)$ and the $\pi_i{}'s$ are $(n \; x \; n)$ matrices of coefficients on lags of Y_t. D_t is a vector of deterministic variables that can contain a linear trend, dummy-type variables, or regressors considered to be fixed and non-stochastic. Finally, e_t is a $(n \; x \; 1)$ vector of independent and identically distributed errors assumed to be normal with zero mean and covariance matrix Ω [i.e., $e_t \sim$ i.i.d. N($\mathbf{0}$, Ω)]. As such, the VARs comprise a system of n equations, where the right-hand side of each equation comprises a common set of lagged and deterministic regressors.

Following Johansen and Juselius (1990), the VAR model provides the basis for co-integration analysis. Adding and subtracting various lags of Y yields an expression for the VARs in first differences, also called a vector error correction model (VECM). That is

$$\Delta Y_t = \pi_0 + \pi Y_{t-1} + \sum_{i=1}^{p-1} \Gamma_i Y_{t-i} + \psi D_t + e_t$$

$$\text{where } \Gamma_i = \sum_{i=1}^{p-1} \theta_i \text{ and } \pi = I_n - \theta_1$$

If π is a zero matrix, then modelling in first differences is appropriate. The matrix π may be of full rank or less than full rank, but of rank greater than zero. When $rank(\pi) = n$, then the original series are not I(1), but in fact I(0); modelling in differences is unnecessary. But, if $0 < rank(\pi) = r < n$, then the matrix π can be expressed as the outer product of two full column rank (n x r) matrices α and β where $\pi = \alpha\beta'$. This implies there are n-r unit roots in πY. The VAR model can then be expressed in reduced rank form. That is

$$\Delta Y_t = \pi_0 + \alpha\beta' Y_{t-1} + \sum_{i=1}^{p-1} \Gamma_i Y_{t-i} + \psi D_t + e_t$$

The matrix β' contains the co-integrating vector(s) and the matrix α has the weighting elements for the rth co-integrating relation in each equation of the VAR. The matrix rows of $\beta' Y_{t-1}$ are normalized on the variable(s) of interest in the co-integrating relation(s) and interpreted as the "long-run" equilibrium condition(s). In this context, the columns of α represent the speed of adjustment coefficients from the "long-run" or equilibrium deviation in each equation. If the coefficient is zero or insignificant in a particular equation, that variable is considered to be weakly exogenous and the VAR can be conditioned on that variable.

Unrestricted model and testing for co-integration
Before conducting the co-integration tests, we need to specify the unrestricted VAR-model properly. That is, determine the appropriate lag-length, test for autocorrelation and non-normality of the residuals. All seven variables are assumed endogenous.

We minimize standard test information criterions to determine the optimal lag length. The Hannan-Quinn and Schwarz criterion (HQ) are minimized when the VAR has one lags, while Akaike's information criterion (AIC) suggests two lags, see Table 4.2. Testing for the importance of all lags confirms the significance of four lags (F-test statistics) and we choose to proceed using a VAR model comprising four lags, corresponding to 1 year.

Table 4.2. Test for lag structure, 1973-2009

	AIC	SC	HQ	F-test [probability]
VAR(4)	-36.31	-30.97	-34.14	**1,39 [0,0451]***
VAR(3)	-36.36	-32.06	-34.61	**1,46 [0,0042]****
VAR(2)	**-36.41**	-33.14	-35.08	**1,86 [0,0000]****
VAR(1)	-36.11	**-33.88**	**-35.20**	**33,27 [0,0000]****

Note: Test when the log-likelihood constant is included. The F-test is a test on the significance of all lags

Sizeable residual outliers are present in some years in the unrestricted VAR, thereby skewing the residual distribution. To reduce this residual misspecification we have included eight dummies mainly concentrated in four episodes: 1) the period around and after the second oil crisis in 1979; 2) the choice to follow a fixed exchange rate regime in 1982/83; 3) the Potato package (*Kartoffel kuren*) introduced 1986/87 including a number of changes affecting the mortgage market such as extending the maturity of mortgages; and 4) the damages following the storm in late 1999 that had a major impact on the path of housing investments the following year. In addition to the dummies, we have included a trend in the model. The inclusion of the dummy variables and the trend results in a significant improvement in the model's fit and a more statistically stable VAR.

Table 4.3 reports summary diagnostic tests on the residuals for the unrestricted VAR with eight lags. The diagnostic tests consist of an F-test for the null hypothesis that there is no residual vector serial correlation; a chi-square test for the null hypothesis of joint normality of the residuals; and finally an F-test for the null hypothesis that there is no residual vector heteroskedasticity. Statistically, the VAR appears well-specified, with no rejections of the null hypothesis from the various test statistics. That is, the VAR residuals appear normal, homoskedastic, and serially uncorrelated.[5]

Table 4.3. Summary diagnostic test statistics for VAR residuals

	Statistic	Value	p-value
Vector AR 1-5 test	F(245,432)	1.037	0.370
Vector Normality test	Chi^2(14)	18.744	0.175
Vector hetero test	F(1624,639)	0.474	1.000

The above analysis indicates that our unrestricted VAR is empirically well-behaved and hence is a suitable starting point for co-integration analysis. The co-integration analysis proceeds in several steps: testing for the existence of co-integration, interpreting and identifying the relationship(s), and inference on the coefficients from theory and weak exogeneity. Testing permits reduction of the unrestricted general model to a final restricted model without loss of information. The trace test statistics in Table 4.4 suggest that our VAR comprises two co-integrating vectors, which is in line with our theoretical demand and supply model.

[5] The diagnostic tests mentioned in the text are vector or system tests.

Table 4.4. Johansen's Co-integration analysis of data

Eigenvalues	Null hypothesis	Trace statistics	p- value
0.388	r=0	202.10**	[0.000]
0.327	r?1	134.47**	[0.002]
0.200	r?2	79.771	[0.188]
0.160	r?3	48.972	[0.464]
0.093	r?4	24.868	[0.795]
0.049	r?5	11.373	[0.849]
0.032	r?6	4.459	[0.678]

Based on the derived equations in the modelling section the two co-integrating vectors are normalized on real house prices and housing investments. The normalized co-integration vectors and the feedback coefficients are presented in Table 4.5.

Table 4.5. Unrestricted VAR model, 1975q2-2009q4

Estimated cointegration vectors β'								
Vector	P	I	Y	UC	PI	H	W	trend
1	1,000	-0.357	0.332	6.007	-0.571	-0.708	0.300	-0.007
2	-0.936	1.000	-0.079	8.748	-1.128	3.196	0.282	-0.007
Feedback coefficients α and their standard errors S.E.								
	α			S.E.				
	1	2		1	2			
P	-0.128	0.040		0.027	0.016			
I	-0.286	-0.155		0.067	0.039			
Y	0.070	0.051		0.078	0.046			
UC	0.009	-0.007		0.004	0.002			
PI	0.006	0.042		0.020	0.012			
H	0.033	-0.028		0.019	0.011			
W	-0.216	0.077		0.051	0.030			

Note: The VAR includes four lags on each variable, a constant, a trend and six dummy variables: D79q1, D83q1, D83q2, D87q1, D88q1, and D00q1.

The long-run co-integrating vectors (or relationships) as they appear are not uniquely identified and hence the standard errors of these vectors cannot be computed. Any linear vector combination forms another stationary vector, so the estimates produced by any particular vector are not necessarily unique. Therefore to achieve identification, it is necessary to impose restrictions on the co-integrating vectors. The restrictions are motivated by our theory. On the demand side equation we impose the restrictions that house prices are not directly affected by building costs, housing investments and the trend. On the supply side, we impose the restrictions that user costs, income and financial wealth do not affect housing investments directly.

Restricted co-integration model and equilibrium values

The result of restricting the just identified model is presented in Table 4.6. All parameters are significant and appear with the right signs. The estimated demand side house price relationship implies that a 1 percent increase real disposable income causes real house price to expand by 0.8 percent, while a similar increase in real financial wealth (excluding pension wealth) brings about a 0.2 percent increase in house prices. The estimated income elasticity is in line with international results, see Girouard *et. al.* (2006) for a survey, and the MONA model (Nationalbanken, 2003), but a great deal below a recent Danish study, which finds an income elasticity closer to 3 Wagner (2005). In line with a number of previous Danish studies we have restricted the housing stock parameter to be the same as to disposable income—though with opposite sign.

Table 4.6. Restricted VAR model, 1975q2-2009q3

Estimated cointegration vectors β' and their standard errors								
Vector	P	I	Y	UC	PI	H	W	trend
1	1.000	-	-0.815	17.811	-	0.815	-0.179	-
	(-)	(-)	(0.413)**	(3.109)**	(-)	(-)	(0.057)**	(-)
2	-1.177	1.000	-	-	1.177	-1.000		0.004
	(-)	(-)	(-)	(-)	(0.153)**	(-)	(-)	(0.001)**
Feedback coefficients α and their standard errors S.E.								
	α			S.E.				
	1	2		1	2			
P	-0.030	0.032		0.010	0.015			
I	-0.135	-0.133		0.027	0.040			
Y	-	-		(-)	(-)			
UC	-	-		(-)	(-)			
PI	-	-		(-)	(-)			
H	-	-		(-)	(-)			
W	-	-		(-)	(-)			

Note: The VAR includes four lags on each variable, a constant, a trend and six dummy variables: D79q1, D83q1, D83q2, D87q1, D88q1, and D00q1.

The (semi)elasticity to user costs implies that a 1 percentage point reduction in user costs (e.g. the real after tax interest rate or property taxes) leads to roughly a 18 percent increase in house price. Other studies on Danish data find a much lower semi-elasticity of about 7-8 percent, see Wagner (2005) and Mona (200x), and international studies have often found even smaller elasticities (Girouard *et. al.,* 2006), though they may not be fully comparable as it sometimes is actual elasticities that are reported. With the current rate for tax deductibility of interest payments (about 33 percent) our estimated semi-elasticity implies that a 1 percentage point increase in the *before-tax* real interest rate leads to a 12 percent reduction in real house prices.[6][7]

In the supply equation, we restrict the parameter to the housing stock to equal 1, implying that the equation can be interpreted as an investment intensity equation (just as in the theoretical section) rather than an investment level equation. The estimated elasticities to house prices and building costs are highly significant, thereby confirming the Tobin's q theory. Even though their parameter

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[6] Over the whole estimation period the tax rate for interest rate deductibility has averaged about 50 percent, which implies that the elasticity to the *before tax* real interest rate averages 8.

[7] Another way to look at the user cost semi-elasticity is to frame it in the more simple asset-pricing approach to the housing market, where the price-to-rent ratio equals the inverse of user costs, see e.g. Girouard *et. al.* (2006). Since housing user costs has averaged about 4 percent over the past 40 years (when including real after tax mortgage rates and property related taxes, but excluding stamp duties and insurance and maintenance costs) a 1 percentage point user costs increase (corresponding to a 20 percent decline in the inverse user costs) would on average have led to a 20 percent reduction in real house prices for a given real rent level. This is higher than the estimated semi-elasticity.

estimates were close when estimated freely we have chosen to restrict them to be fully identical, so it is their relative price that drives the investment intensity. The coefficients indicate an almost one-to-one relationship between the house prices to building costs ratio on the one hand and the housing investment intensity on the other.

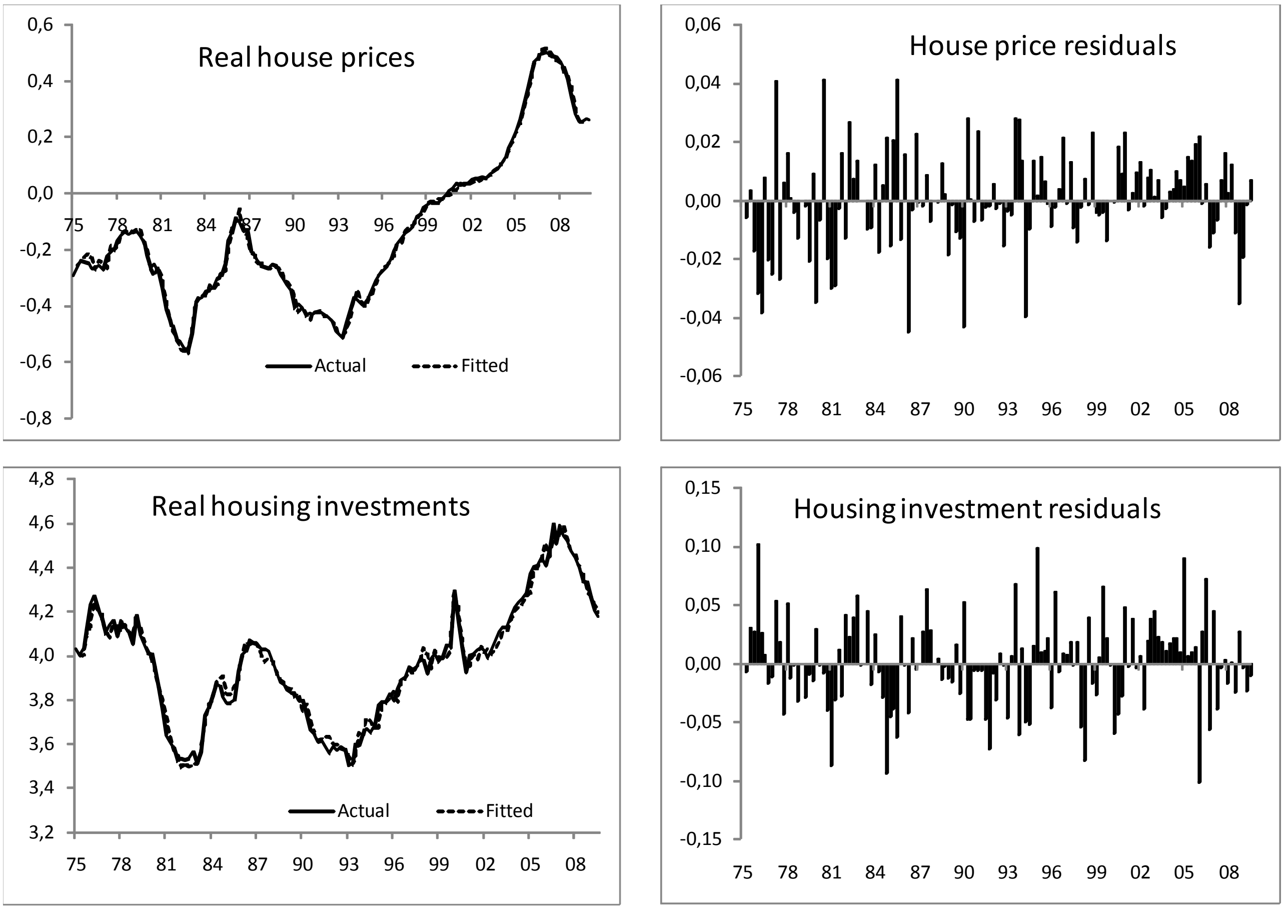
The negative time trend is also highly significant, suggesting that investment intensity over time is decreasing given the development in the house price to building cost ratio. The could mean that either the time trend is capturing a declining trend in the depreciation rate of the housing stock, suggesting that less investment is needed to maintain the housing stock (e.g. if the quality of investments have improved without being adequately captured in the investment data) or that the increase in the house price to building cost ratio is overstated in the data (e.g. since we do not (adequately) include the price of land and building plots we could be understating the true building costs if land prices have increased faster than other building costs).

The signs of the loading or speed of adjustment parameters have the appropriate signs. When house prices are above (below) equilibrium they tend to re-correct downward (upward) by 3 percent per quarter, while housing investments adjust upward (downward) by 3 percent per quarter. This point to a rather strong inertia in the house price equation as real prices adjusts slowly to dis-equilibriums. The feedback is much faster when housing investments deviate from their equilibrium value as both house prices investments adjust by 13 percent per quarter.[8]

A plot of the actual and fitted variables and the residual points to a good model fit (see Figure 4.1). The small cluster of positive residuals during 2005-06 in the price equation suggests that additional factors may have played a role in these boom years. Additional behavioural factors could come from less household risk-aversion after a long period of constantly declining unemployment rates, the introduction of interest-only mortgages, and possibly increased lending willingness by financial institutions approving loans with less collateral and smaller disposable income margins. The latter may have been affected by booming pension wealth, which cannot be used as mortgage collateral and therefore is not included in our financial wealth variable. Likewise the large negative residuals end-2008 to early-2009 may reflect negative confidence effects from the economic and financial crisis.

[8] The feed-back coefficients to most variables other than house price and housing investment appear small and insignificant, so we test the joint hypothesis of setting those equal to zero. This test cannot be rejected, and as such they seem weakly exogenous with respect to house prices and real housing investments.

Figure 4.1. Actual and Fitted Values and Residuals



Note: All variables in charts are measured in natural logarithm.

5. Long-run Equilibrium and Medium Term Scenarios

Simulation of long-run equilibrium values

We can use the estimated model to simulate long run equilibrium values for house prices and investments. To do that we substitute the two long-run, co-integrating, equations into each other to obtain our equilibrium equations for house prices and investments. However, for a fixed depreciation rate the steady state growth rate in housing investments and housing stock is the same, implying constant steady state investment intensity. Thus, we can rearrange the Tobin's q supply equation to show that in steady state equilibrium house price growth is determined from the supply side. This rearrangement implies that house price growth is determined by growth in building costs and in the trend:

$$\partial logP_t = \partial logPI_t + 0{,}003 \cdot \partial t$$

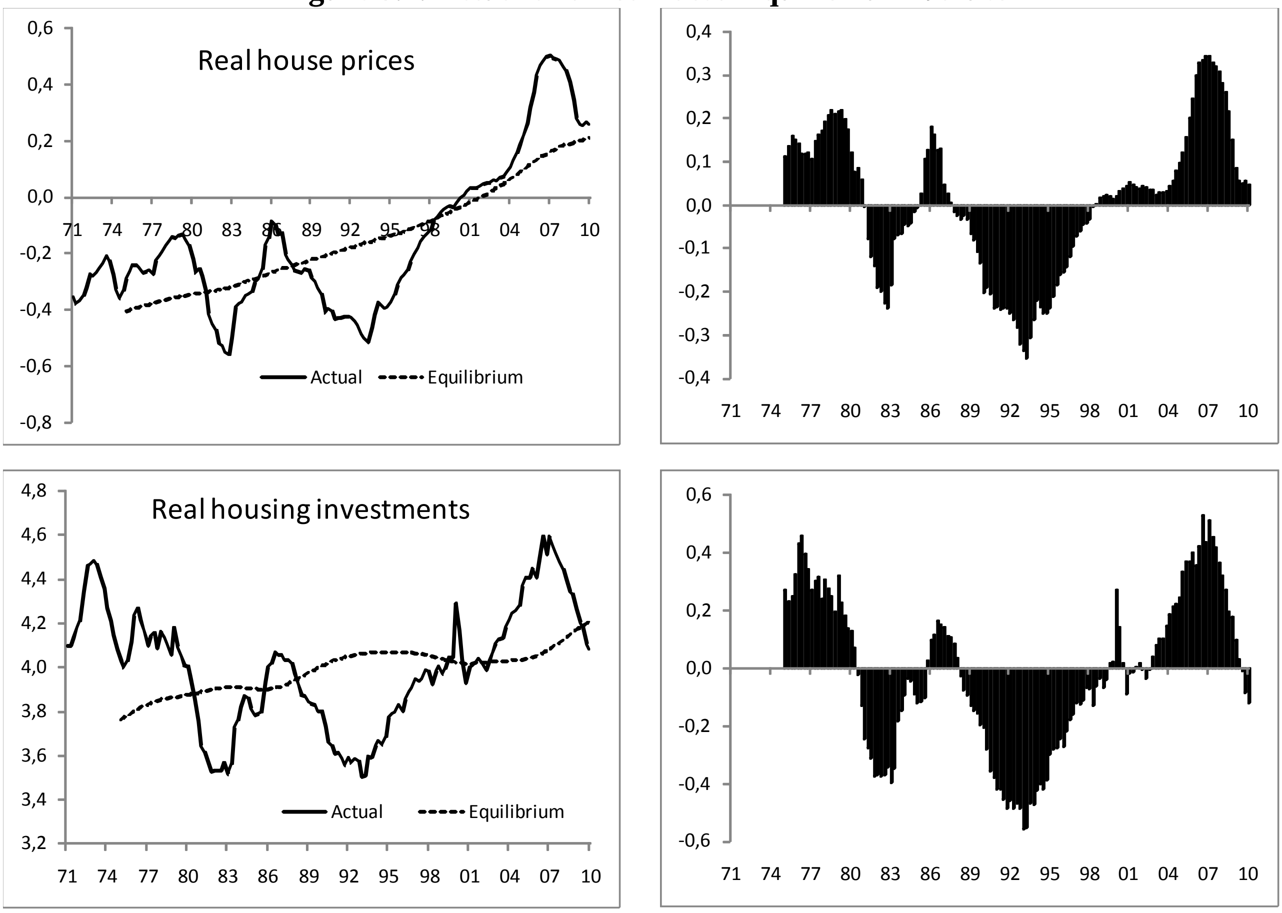
Thus, an increase in building costs leads to an equivalent increase in steady state house prices. In addition, the time trend captures additional real house price increases not stemming from higher building costs (e.g. housing quality improvements or land costs not adequately captured by the statistics). We can insert this price equation into the demand equation to determine growth in the housing stock and investments:

$$\partial logH_t = \partial logY_t + 0.22 \cdot \partial logW_t - 21.86 \cdot \partial UC_t - 1.23 \cdot \partial logP_t$$

We see that an increase in disposable income leads to an equivalent increase in the housing stock. The impact is somewhat smaller for financial wealth, while the impact from user costs is sizeable.

To construct the equilibrium values we smooth the independent series and remove short term fluctuations. For disposable income, financial wealth, and building costs we compute trend values by applying the standard Hodrick-Prescott filtering procedure. Housing user costs have over the past 40 years tended to swing back and forth around roughly 4 percent (see also Figure 2.3) and so we have decided to use this as our benchmark for user costs, even though it reflects a multiple of changes in tax rates, interest rates and inflations expectations.

Figure 5.1. Actual and Estimated Equilibrium Values



Note: All variables in charts are measured in natural logarithm.

The simulated outcomes indicate that the recent declines in house prices and investments have pushed them close to or even below their equilibrium value, see Figure 5.1. However, reaching equilibrium does not in itself mean that they will stop falling, even though house prices seem to have stabilised. In previous periods of house price depreciations they have tended to undershoot equilibrium significantly due to seemingly strong inertia (as indicated by the low speed of adjustment parameters to house price deviations), although the large undershooting from the late 1980s to mid 1990s was triggered by a sizeable cut in the tax rate for deduction of mortgage interest payments. However, the time period before house prices bottom-out relative to the equilibrium price can take 3-5 years (that is, the time period after actual real prices have passed their equilibrium in a bust period). Since the estimated equilibrium price itself is growing the potential downward adjustment in real house prices is smaller than what the "normal" undershooting may indicate. Similarly, housing investments can potentially also undershoot equilibrium significantly.

House price developments in the period ahead

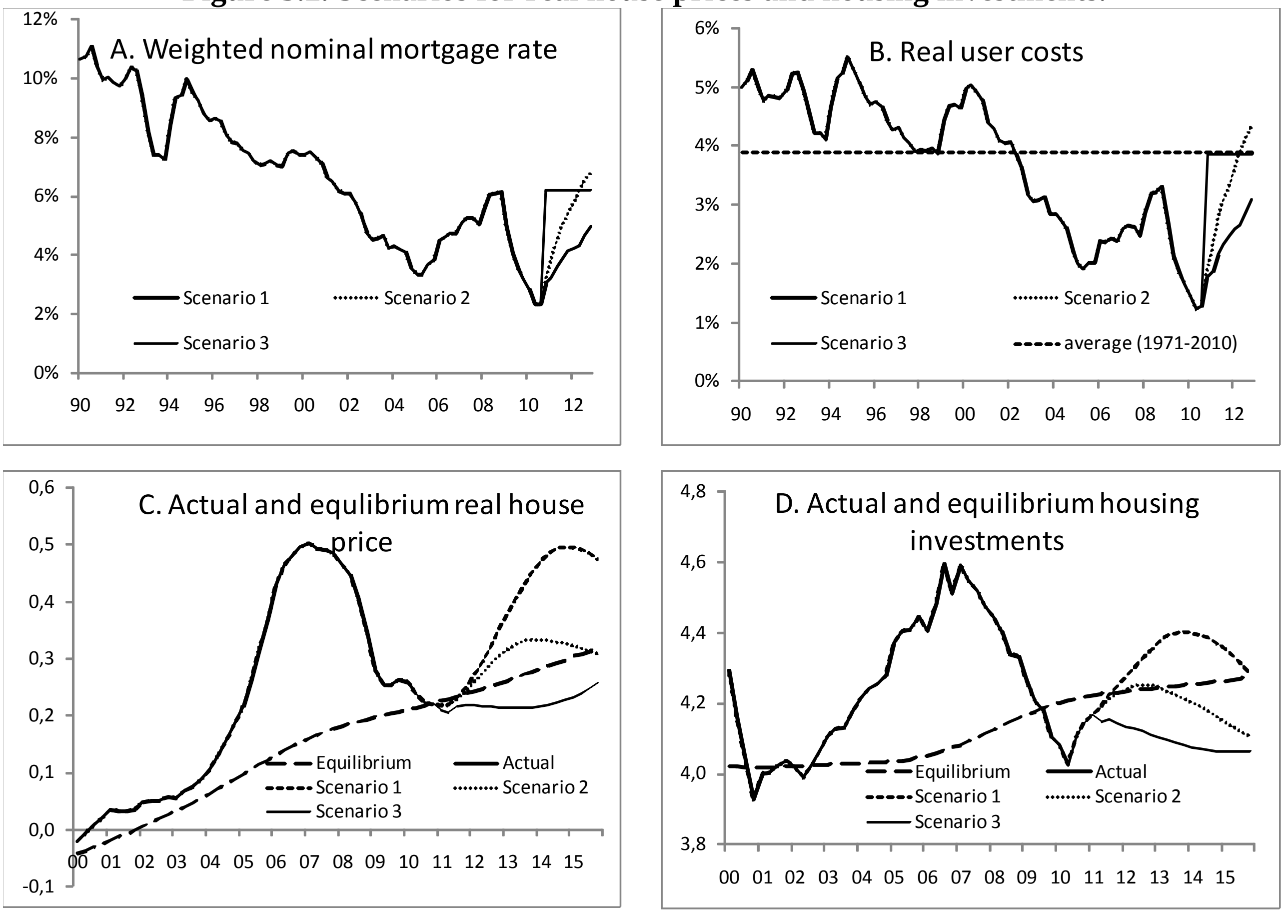
We can use the estimated short run dynamics in the VECM to construct medium term scenarios for developments in house prices and investments. To do that, we have created two main (and one extreme benchmark) scenarios for mortgage rate developments. To keep things simple we leave income growth and wealth assumptions—which broadly follow the Economic Survey August 2010 (see Finansministeriet, 2010)— unchanged between the two main scenarios This is obviously a simplification as different mortgage rates will alter (and may have been altered by) income and wealth growth developments.

The average mortgage rate has already declined from just above 6 percent at end-2008 to only 2¼ percent in the third quarter of 2010 due to sharp declines in particularly the short term interest rate, but also the long term interest rates. In both main scenarios we assume the average mortgage rate remains more or less at this level until mid 2011. In scenario 1, the rate then gradually increases to almost 5 percent by end 2012 due to assumed progressive monetary policy tightening. This implies a long term nominal mortgage of 6 percent and a short term rate slightly below 4 percent, and so the historical average spreads between the discount rate and the mortgage rates are maintained. In scenario 2, the average mortgage rate gradually increases to almost 6 percent (long term rate at 7½ percent and a short term rate at 4½ percent). In this scenario the assumed monetary tightening (the central bank's discount rate increases to 4 percent instead of 3½ percent by end 2012) is slightly higher and the spreads between the discount rate and mortgage rates are kept at their higher average 2009 level. However, we compare those two scenarios with an extreme (and clearly unrealistic) third benchmark scenario where the average mortgage rate is assumed to jump to 6¼ percent in fourth quarter 2010 so total user costs are at their historical average.

Both main scenarios imply that the stabilisation of real house prices—following the sharp decline from early 2007 to mid 2009—is shifting to moderate increases thanks to historically low mortgage rates. Even the expected monetary tightening starting mid 2011 does not in the short term stem the house prices increases already in train as real user costs remain below the historical average (see Figure 5.2). The real difference between the two main scenarios is the pace and degree of real house price increases in the short term. For instance, between first quarter 2010 and fourth quarter 2012 real house prices increases by 9 percent in scenario 1 and 5 percent in scenario 2. However, it cannot be ruled out that the model does not fully capture financial market tensions in the aftermath of the financial crisis (including tighter lending standards) and so the scenarios may overstate actual short term increases. It is only in the extreme benchmark scenario 3 that we actually witness a real house price decline in the short term as the sharp hike in real user costs outweighs the lagged effects of low user costs and strong house price inertia. In the medium term to 2015 real house prices readjusts towards the equilibrium value in all three scenarios, implying real house price falls in the two main scenarios.

Meanwhile, the downward adjustment in housing investments is also coming to an end. As a result of the exceptionally low mortgage rates the simulations imply a sharp rebound in investments in the second half of 2010 and continue at a strong pace through 2012 in scenario 1, although as with house prices the short term effects may be overstated given confidence effects. In scenario 2 the growth rate tapers off somewhat more during 2011-12. Both scenarios imply declining investments in the medium term as a response to increasing user costs.

Figure 5.2. Scenarios for real house prices and housing investments.



Note: Values in Figure C and D are in natural logarithm.

6. Conclusion

House prices have come down in Denmark after a long period of high growth. Declining house prices has been particularly evident in Copenhagen for a while, but real prices now appear to have stabilised. The previous recent trough-to-peak house price increases were high in both a historic and an international perspective. In this paper we assess whether current house price and housing investment levels are in line with economic fundamentals and long term trends.

By using multivariate econometric techniques to estimate a demand-supply housing market model our analysis indicates that real house prices in 2004 started to climb well above their long-term equilibrium level and peaked in 2007 more than 30 percent above equilibrium. The recent significant downward price adjustments—following the financial and economic crisis—have all but removed this disequilibirum and real house prices are estimated to be close to their equilibrium level. The proximity to equilibrium and the current low interest rate levels may have contributed to the stabilisation of house prices. Housing investments may now even be below an estimated equilibrium level and could soon start to rise again.

Our analysis also suggests that the real house price appreciation in general were supported by economic fundamentals such as strong growth in real disposable income and financial wealth, and real mortgage rates falling well below historical averages. However, only a small part of the price increases during 2004-07 appears to have gone beyond the fundamentals included in our estimated model and could reflect increasing risk appetite by households and financial institutions as a response to previous price gains (price inertia), low unemployment, and the introduction interest-only mortgages.

The findings of this paper are in line with the existing literature indicating that the recent increases have taken house prices above their long term trend in many advanced countries. Besides eliciting some insights about recent house price developments in the Danish housing markets, our paper complements this literature principally by setting up a well defined theoretical framework for a housing market model as well as estimating the model using a multivariate econometric approach.

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Appendix A.1. Solving the Theoretical Model

Demand side

The representative household derives utility by maximising its utility function in all future periods discounted by the rate of time preference, β. The consumer derives utility from consumption of a composite good, c_t, and housing services, h_t:

$$\max_{c,h} E_0 \sum_{t=0}^{\infty}(1+\beta)^{-t}u(c_t, h_t) \qquad (A.1.1)$$

The representative household is subject to a budget constraint that holds each period:

$$W_t = (1+r)W_{t-1} + y_t - c_t - p_t h_t$$

Where W_t is real financial wealth (including housing equity), r is the constant real riskless return on the equilibrium asset portfolio, y_t is real endowment income, and p_t is the real house price (the price of c_t is numeriare). Real housing consumption, h_t, equals the user cost of housing, uc_t, times the housing stock, *H*; $h_t = uc_t \cdot H_t$. Expressing the budget constraint in present value terms yields:

$$\sum_{t=0}^{\infty}(1+r)^{-t}c_t = W_0 + \sum_{t=0}^{\infty}(1+r)^{-t}(y_t - p_t h_t) \qquad (A.1.2)$$

Hence, the net present value of non-housing consumption equals initial wealth, W_0, plus the net present value of income minus the housing consumption. Given the budget constraint we can solve the representative household's maximisation problem via the usual Lagrange function:

$$L = (1+\beta)^{-t}u(c_t, h_t) + \lambda[W_0 + \sum_{t=0}^{\infty}(1+r)^{-t}(y_t - p_t h_t - c_t)] \qquad (A.1.3)$$

The first-order conditions to the household's maximisation problem imply the following relationships between marginal utilities u_c' and u_H':

$$L'_c = (1+\beta)^{-t}u_c'(c_t, h_t) - \lambda(1+r)^{-t} = 0$$

$$L'_h = (1+\beta)^{-t}u_h'(c_t, h_t) - \lambda(1+r)^{-t} \cdot p_t = 0$$

Dividing the two first order conditions with each other gives the usual intra-temporal relationship between marginal utilities showing that the marginal rate of substitution (MRS) between non-housing and housing consumption is equal to the relative prices (since the price of c_t is numeraire):

$$\frac{u_h'(c_t, h_t)}{u_c'(c_t, h_t)} = p_t \qquad (A.1.4)$$

As the representative consumer also maximizes utility across time we calculate the development of consumption and household consumption over time. The standard Euler-equations can be derived from the first order conditions by dividing the conditions in two periods. We assume the interest rate to be constant:

$$\frac{L_c}{L_{c+1}} = \frac{(1+\beta)^{-t}u_c'(c_t, h_t) - \lambda(1+r)^{-t}}{(1+\beta)^{-(t+1)}u_c'(c_{t+1}, h_{t+1}) - \lambda(1+r)^{-(t+1)}} = 0 \quad \Leftrightarrow \quad \frac{u_c'(c_t, h_t)}{u_c'(c_{t+1}, h_{t+1})} = \frac{1+r}{1+\beta} \qquad (A.1.5)$$

And in the same way the development of household consumption is calculated, where π_t is the real house price growth:

$$\frac{u_h'(c_t,h_t)}{u_h'(c_{t+1},h_{t+1})} = \frac{1+r}{1+\beta}\ (1+\pi_t) \qquad (A.1.6)$$

We assume the following functional form for the utility function:

$$u(c_t,h_t) = \frac{\gamma \cdot c_t^{1-\theta} + (1-\gamma)\cdot h_t^{1-\theta}}{1-\theta},$$

By inserting in the MRS equation (A.1.4) we get the following expression for house prices:

$$p_t = \frac{1-\gamma}{\gamma} \cdot \left(\frac{c_t}{h_t}\right)^{\theta} \qquad (A.1.7)$$

Recalling that $h_t = uc_t \cdot H_t$ and taking the logarithm yields:

$$\log p_t = \theta \cdot (\log c_t - \log uc_t - \log H_t) + \log\left(\frac{1-\gamma}{\gamma}\right) \qquad (A.1.8)$$

By using the functional form for the utility function we can rewrite the Euler-equations (A.1.5) and (A.1.6) and arrive at the following expressions for c_t and h_t:

$$c_t = c_0\left(\frac{1+r}{1+\beta}\right)^{\frac{1}{\theta}t} \quad \text{and} \quad h_t = h_0\left(\frac{(1+r)(1+\pi_t)}{1+\beta}\right)^{\frac{1}{\theta}t}$$

Next, incorporating the rewritten Euler equations into the budget constraint implies:

$$c_0 \sum_{t=0}^{\infty}\left(\frac{1+r}{1+\beta}\right)^{\frac{1}{\theta}t}(1+r)^{-t} = W_0 + \sum_{t=0}^{\infty}(1+r)^{-t} y_t - h_0 \sum_{t=0}^{\infty}\left(\frac{(1+r)(1+\pi_t)}{1+\beta}\right)^{\frac{1}{\theta}t}(1+r)^{-t} p_t$$

We can rewrite this to a simpler expression:

$$c_0 = \mu\,(W_0 + \hat{y}_0 - \varphi_0 p_0 h_0) \qquad (A.1.9)$$

where $\hat{y}_0 = \sum_{t=0}^{\infty}(1+r)^{-t} y_t$ is the net present value of future income streams, $\mu = \left(\sum_{t=0}^{\infty}\left(\frac{1+r}{1+\beta}\right)^{\frac{1}{\theta}t}(1+r)^{-t}\right)^{-1}$ is the propensity to consume and $\varphi_0 p_0 = \sum_{t=0}^{\infty}(1+\beta)^{-(1/\theta)t} \cdot (1+r)^{((1+\theta)/\theta)t} \cdot (1+\pi_t)^{((1-\theta)/\theta)t}\, p_0$ since we can write $p_t = p_0 \cdot (1+\pi_t)^t$.

Expressing the budget constraint (A.1.9) in log terms yields:

$$\log c_0 = \log \mu_0 + C_W \cdot \log W_0 + C_Y \cdot \log \hat{y}_0 - C_h \cdot (\log \varphi_0 + \log p_0 + \log h_0) \qquad (A.1.10)$$

where $C_W = \frac{W}{W+\hat{y}-\varphi p h}$, $C_y = \frac{\hat{y}}{W+\hat{y}-\varphi p h}$ and $C_h = \frac{\varphi p h}{W+\hat{y}-\varphi p h}$.

Inserting this into the MRS equation (A.1.8) gives us the demand side house price equation:

$$\log p_0 = \sigma \log W_0 + \omega \log \hat{y}_0 - \rho(\log H_0 + \log uc_0) + \vartheta_0 \qquad (A.1.11)$$

where $\sigma = \frac{\theta C_W}{1+\theta C_h}, \omega = \frac{\theta C_y}{1+\theta C_h}, \rho = \frac{\theta(1+C_h)}{1+\theta C_h}$ and $\vartheta_0 = \frac{\theta}{1+\theta C_h} log \mu_0 - \frac{\theta C_h}{1+\theta C_h} log \varphi_0 + \frac{1}{1+\theta C_h} log\left(\frac{1-\gamma}{\gamma}\right)$.

The equation shows the standard result that house prices depend negatively on user costs and housing stock, while positively on wealth and disposable income, see Barot and Yang (2002).

Supply side

To model the supply side we follow Brøchner (1992) and Poterba (1984) who use an intertemporal capital stock approach where the representative firm chooses the optimal amount of investment by maximising the net present value of profits:

$$\max_{H,I} \Pi = \sum_{t=0}^{\infty}(1+r)^{-t}[p_t^H H_t - p_t^I I_t - C(I,H)] \qquad (A.1.12)$$

The representative firm maximises subject to the evolution of housing stock:

$$H_{t+1} = (1-\delta)H_t + I_t$$

Profits are given by the value of housing, $p_t^H H_t$, minus the costs of investment which is divided into direct investment costs, $p_t^I I_t$, and some convex adjustment costs, $C(I,H) = ½\beta\frac{I^2}{H}$, which is homogenous of degree one. The investment costs are strictly convex in I: $C_I{}'(I,H) > 0$, $C_I{}''(I,H) > 0$ and $C_H{}'(I,H) < 0$.

Given the budget constraint we can solve the representative firm's maximisation problem via the usual Lagrange function:

$$L = \sum_{t=0}^{\infty}(1+r)^{-t}[p_t^H H_t - p_t^I I_t - C(I,H)] + \sum_{t=0}^{\infty}\lambda[(1-\delta)H_t + I_t - H_{t+1}] \qquad (A.1.13)$$

Defining $q_t = (1+r)^t\lambda$ we can rewrite (A.1.13):

$$L = \sum_{t=0}^{\infty}(1+r)^{-t}[p_t^H H_t - p_t^I I_t - C(I,H) + q_t((1-\delta)H_t + I_t - H_{t+1})] \qquad (A.1.14)$$

Solving the problem yields the first order conditions L_I and L_H:

$$L_I = (1+r)^{-t}(-p_t^I - C_I'(I,H) + q_t) = 0 \quad \Leftrightarrow \quad C_I'(I,H) = q_t - p_t^I \qquad (A.1.15)$$

$$L_H = (1+r)^{-t}\left(p_t^H - C_H'(I,H) + q_t(1-\delta)\right) - (1+r)^{-(t-1)}q_{t-1} = 0$$

$$\Leftrightarrow p_t^H = (1+r)q_{t-1} + C_H'(I,H) - q_t(1-\delta) \qquad (A.1.16)$$

$$lim(1+r)^{-t}q^t H^t - 0 \qquad (A.1.17)$$

Equation (A.1.15) states that the representative firm invests to the point where the price of investment (direct and adjustment costs) equals the value of housing (i.e. the shadow price q_t is equal to the market value of houses relative to the replacement value). Equation (A.1.17) is the standard transversality condition stating that the present value of the housing stock at infinity is zero.

From the first order conditions it is possible to derive the firms' optimal amount of investment. By solving equation (A.1.16) forward and solving for q_0 we get the following expression, where $p_t^H - (1+\pi_t)^t p_0^H$.

$$q_t = \frac{1}{1-\delta}\left((1+r)q_{t-1} - p_t^H + C_H'(I,H)\right) \Leftrightarrow \cdots \quad \Leftrightarrow q_0 = \frac{1}{1-\delta}\sum_{t=0}^{\infty}\left(\frac{1-\delta}{1+r}\right)^t \left(p_t^H - C_H'(I,H)\right)$$
$$= \frac{1}{1-\delta}\sum_{t=0}^{\infty}\left(\frac{(1-\delta)(1+\pi_t)}{(1+r)}\right)^t p_0^H - \frac{1}{1-\delta}\sum_{t=0}^{\infty}\left(\frac{(1-\delta)}{(1+r)}\right)^t C_H'(I,H) \qquad (A.1.18)$$

This states that q_0 is the negative net present value of the house prices. Inserting (A.1.18) into (A.1.15), rearranging and log-linerize around one we get an expression for the firm's optimal investment at time zero (recall: $C_I'(I,H) = \beta\frac{I}{H}$) and $C_H'(I,H) = -½\beta\left(\frac{I}{H}\right)^2$:

$$\beta\left(\frac{I}{H}\right) - \frac{\sigma}{2(1-\delta)}\beta\left(\frac{I}{H}\right)^2 = \frac{1}{1-\delta}\sum_{t=0}^{\infty}\left(\frac{(1-\delta)(1+\pi)}{1+r}\right)^t p_0^H - p_0^I = \left(\frac{\mu}{1-\delta}\right)p_0^H - p_0^I \quad \text{(A.1.19)}$$

where $\mu = \left(1 - \frac{(1-\delta)(1+\pi)}{(1+r)}\right)^{-1}$ and $\sigma = \left(1 - \frac{(1-\delta)}{(1+r)}\right)^{-1}$.

Log-linerizing around one the left hand side of (A.1.19) – recall log(a)=a-1 if a is small:

$$log\left(\beta\left(\frac{I}{H}\right) - \frac{\sigma}{2(1-\delta)}\beta\left(\frac{I}{H}\right)^2\right) = \log\left(\beta\left(\frac{I}{H}\right)\left(1 - \frac{\sigma}{2(1-\delta)}\left(\frac{I}{H}\right)\right)\right)$$
$$= log(\beta) + log\left(\frac{I}{H}\right) + log\left(1 - \frac{\sigma}{2(1-\delta)}\left(\frac{I}{H}\right)\right)$$
$$\approx log(\beta) + \left(\frac{I}{H}\right) - 1 + log\left(1 - \frac{\sigma}{2(1-\delta)}\right) - \frac{1}{\frac{2(1-\delta)}{\sigma} - 1}\left(\left(\frac{I}{H}\right) - 1\right)$$
$$= log(\beta) + log\left(1 - \frac{\sigma}{2(1-\delta)}\right) + \left(1 - \frac{1}{\frac{2(1-\delta)}{\sigma} - 1}\right) log\left(\frac{I}{H}\right)$$

Log-linerizing around one on the right hand side of (A.1.19):

$$log\left(\left(\frac{\mu}{1-\delta}\right)p_0^H - p_0^I\right) \approx \log\left(\frac{\mu}{1-\delta} - 1\right) + \frac{1}{1 - \frac{1-\delta}{\mu}} log(p_0^H) - \frac{1}{\frac{\mu}{1-\delta} - 1} log(p_0^I)$$

Collecting terms and isolating:

$$log\left(\frac{I}{H}\right) - \frac{1}{\varphi}\left(\rho + \frac{\omega}{\omega-1} log(p_0^H) - \frac{1}{\omega-1} log(p_0^I)\right) \qquad \text{(A. 1.20)}$$

Where $\rho = \left(\log\left(\frac{\mu}{1-\delta} - 1\right)\right) - log(\beta) - log\left(1 - \frac{\sigma}{2(1-\delta)}\right)$, $\omega = \frac{\mu}{1-\delta} > 1$ and $\varphi = 1 - \frac{1}{\frac{2(1-\delta)}{\sigma} - 1} > 0$.

Equation (A.1.20) is the supply function in our modelling framework and it states that housing investment in equilibrium depends positively on the price and negatively on the investment costs.

Appendix A.2. Choice of Variables and Data

Table A.2.1 Variables and sources

	Construction	Source
Variables:		
Real house prices in log (P)	Nominal price index deflated by consumer price index.	MONA
Real housing investments in log (fih)		MONA
Real household disposable income in log (fydph)	Real disposable income (FYDP) subtracted gross operating surplus and deflated by consumer price index.	MONA
Real financial wealth in log (w)	See Overgaard (2008) for details. Deflated by consumer price index	Nationalbanken.
Real user costs (UC)	For the period 1986-2010 a weighted nominal mortgage rate is computed by using Realkreditråd interest rate data for a 30-year fixed rate mortgage and a 1-year adjustable rate mortgage. The weights are the outstanding loan stock shares of those mortgages. Before 1986 we use the interest rate in the MONA databank. Expected inflation is simply computed by HP filtering actual inflation (private consumption deflator). For the period before 1982 we reduced the filtered series by a factor 0.6. Between 1982 and 1992 this factor linearly attains 1 and remains at 1 from 1992 and onwards. The procedure is in the spirit of the Mona approach to compute an expected inflation series. On SKAT's website www.SKAT.dk we have found tax rate data for mortgage rate deductibility and property *value* taxation (lejeværdi skat). Both series date back to 1970. The average property tax rate (land tax) is calculated by dividing the total tax revenue with the value of the housing stock.	SKAT, Statistics Denmark, Mona and Realkreditrådet
Real building costs in log (pih)	From 1992-2008 we use a weighted average of the building cost index and the price on land (plots less than 2000 m2). Before 1992 we use the housing investment deflator. Deflated by the consumer price index.	MONA and Statistics Denmark
Real housing stock log (fwh)	Housing wealth in Overgaard (2008) deflated with the house price index.	MONA and Nationalbanken.

Source: Statistics Denmark, The Mona databank (Nationalbanken), and Realkreditrådet.